Integrals

Assertion & Reason Type Questions

Directions: In the following questions, each question contains Assertion (A) and Reason (R). Each question has 4 choices (a), (b), (c) and (d) out of which only one is correct. The choices are:

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)

b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)

c. Assertion (A) is true and Reason (R) is false

d. Assertion (A) is false and Reason (R) is true

Q1. Assertion (A):

$$\int \sin 3x \cos 5x \, dx = \frac{-\cos 8x}{16} + \frac{\cos 2x}{4} + C$$

Reason (R): 2 cos A sin B = sin (A + B) - sin (A - B)

Answer : (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)

Q2.

Let $I = \int \frac{\sqrt{\cos 2x}}{\sin x} dx$

Assertion (A):

$$I = \ln\left[\left(\frac{t}{1+\sqrt{1-t^2}}\right)\left(\frac{\sqrt{2}+\sqrt{1-t^2}}{\sqrt{2}-\sqrt{1-t^2}}\right)^{\frac{1}{\sqrt{2}}}\right] + C$$

where, $t = \tan x$

Reason (R): $\int \cot \theta \, d\theta = \log|\sin \theta| + C$

Answer : (d) Assertion (A) is false and Reason (R) is true

Q3. Let F(x) be an indefinite integral of sin² x.

Assertion (A): The function F(x) satisfies F(x + π) = F(x) for all real x.

Reason (R): $\sin^2(x + \pi) = \sin^2 x$ for all real x.

Answer : (d) Assertion (A) is false and Reason (R) is true

Q4.

Assertion (A): $\int_2^8 \frac{\sqrt{10-x}}{\sqrt{x}+\sqrt{10-x}}\,dx = 3$

Reason(R): $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

Answer : (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)

Q5.

Assertion (A): $I = \int_0^1 \frac{dx}{\sqrt[3]{1+x^3}} = \int_0^{2^{-1/3}} \frac{dt}{1-t^3}$

Reason (R): The integrand of the integral I becomes rational by the substitution $t = \frac{x}{\sqrt[3]{1+x^3}}$.

Answer : (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)

Q6.

Assertion (A): $\int_0^{\pi} \cos 6x \cdot \cos 6x\,dx = \frac{\pi}{32}$

Reason (R): $\int_0^{\pi} \cos mx \cos nx\,dx = 0,$

$m \neq n,\ m, n \in Z.$

Answer : (d) Assertion (A) is false and Reason (R) is true

Q7.

Assertion (A):

$\int_{-\pi/3}^{\pi/3} \frac{(3+4x^3)\,dx}{2-\cos\left(|x|+\frac{\pi}{3}\right)} = 4\sqrt{3}\tan^{-1}\left(\frac{1}{2}\right)$

Reason (R): $\int_{-a}^{a} f(x)\,dx = \int_0^a [f(x) + f(-x)]\,dx$

Answer : (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)

Q8.

Assertion (A): $\int_0^{2\pi} \sin^3 x\, dx = 0$

Reason (R): $\sin^3 x$ **is an odd function.**

Answer : (b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)

Assertion (A): $\int \frac{dx}{x^2+2x+3} = \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x+1}{\sqrt{2}}\right)+c$

Reason (R): $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c$

Ans. Option (A) is correct.

Explanation:

$$\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c.$$

This is a standard integral and hence R is true.

$$\int \frac{dx}{x^2+2x+3} = \int \frac{dx}{(x+1)^2+(\sqrt{2})^2}$$
$$= \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x+1}{\sqrt{2}}\right)+c$$

Hence A is true and R is the correct explanation for A.

Assertion(A): $\int e^x[\sin x - \cos x]\,dx = e^x \sin x + C$

Reason (R): $\int e^x[f(x)+f'(x)]\,dx = e^x f(x)+c$

Ans. Option (D) is correct.

Explanation:

$$\int e^x[f(x)+f'(x)]dx = \int e^x f(x)dx + \int e^x f'(x)dx$$
$$= f(x)e^x - \int f'(x)e^x dx + \int f'(x)e^x dx$$
$$= e^x f(x)+c$$

Hence R is true.

$$\int e^x(\sin x - \cos x)dx = e^x(-\cos x)+c$$
$$= -e^x \cos x + c$$
$$\left[\because \frac{d}{dx}(-\cos x) = \sin x\right]$$

Hence A is false.

Assertion (A): $\int x^x(1+\log x)dx = x^x + c$

Reason (R): $\frac{d}{dx}(x^x) = x^x(1+\log x)$

Ans. Option (A) is correct.

Explanation : Let $y = x^x$

$\Rightarrow \log y = x\log x$

Differentiating w.r.t. x

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = x\left(\frac{1}{x}\right)+\log x(1)$$
$$\frac{dy}{dx} = y(1+\log x)$$
$$= x^x(1+\log x)$$

Hence R is true.

Since $\frac{d}{dx}(x^x) = x^x(1+\log x)$

$$\int x^x(1+\log x)dx = x^x + c$$

Using the concept of anti-derivative, A is true. R is the correct explanation for A.

Assertion (A): $\int x^2 dx = \frac{x^3}{3}+c$

Reason (R): $\int e^{x^2}dx = e^{x^3/3}+c$

Ans. Option (C) is correct.

Explanation:

Since $\int x^n\,dx = \frac{x^{n+1}}{n+1}+c,$

$$\int x^2 dx = \frac{x^{2+1}}{2+1}+c$$
$$= \frac{x^3}{3}+c$$

$\therefore$ A is true.

$\int e^{x^2}dx$ is a function which can not be integrated.

$\therefore$ R is false.

Assertion (A): $\int_0^{\pi/2} \frac{\cos x}{\sin x + \cos x}dx = \frac{\pi}{4}$

Reason (R): $\int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x}dx = \frac{\pi}{4}$

Ans. Option (A) is correct.

Explanation:

Let $$I = \int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x} dx \quad ...(i)$$

$$\int_0^a f(x)dx = \int_0^a f(a-x)dx$$

$$\therefore \quad I = \int_0^{\pi/2} \frac{\sin\left(\frac{\pi}{2}-x\right)dx}{\sin\left(\frac{\pi}{2}-x\right)+\cos\left(\frac{\pi}{2}-x\right)}$$

$$I = \int \frac{\cos x}{\cos x + \sin x} dx \quad ...(ii)$$

Adding equations (i) + (ii),

$$\Rightarrow \quad 2I = \int_0^{\pi/2} \frac{\sin x + \cos x}{\sin x + \cos x} dx$$

$$= \int_0^{\pi/2} 1\, dx$$

$$= [x]_0^{\pi/2}$$

$$= \frac{\pi}{2}$$

$$\therefore \quad I = \frac{\pi}{4}$$

Hence R is true.

From (ii), A is also true.

R is the correct explanation for A.

Assertion (A): $\int_{-3}^{3} (x^3+5)dx = 30$

Reason (R): $f(x) = x^3 + 5$ is an odd function.

Ans. Option (C) is correct.

Explanation:

Let
$$f(x) = x^3+5$$
$$f(-x) = (-x)^3+5$$
$$= -x^3+5$$

$f(x)$ is neither even nor odd. Hence R is false.

$$\int_{-3}^{3} x^3\, dx = 0 \qquad [\because x^3 \text{ is odd}]$$

$$\int_{3}^{3} 5dx = 5[x]_{-3}^{3} = 30$$

$$\therefore \int_{-3}^{3} (x^3+5)dx = 0+30 = 30$$

Hence A is true.

Assertion (A): $\frac{d}{dx}\left[\int_0^{x^2} \frac{dt}{t^2+4}\right] = \frac{2x}{x^4+4}$

Reason (R): $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c$

Ans. Option (A) is correct.

Explanation:

$$\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c.$$

This is a standard integral and hence true.

So R is true.

$$\int_0^{x^2} \frac{dt}{t^2+4} = \left[\frac{1}{2}\tan^{-1}\left(\frac{t}{2}\right)\right]_0^{x^2}$$

$$= \frac{1}{2}\tan^{-1}\left(\frac{x^2}{2}\right)$$

$$\frac{d}{dx}\left[\int_0^{x^2} \frac{dt}{t^2+4}\right] = \frac{d}{dx}\left[\frac{1}{2}\tan^{-1}\left(\frac{x^2}{2}\right)\right]$$

$$= \frac{1}{2}\times\frac{1}{1+\frac{x^4}{4}}\times\frac{2x}{2}$$

$$= \frac{x}{2}\times\frac{4}{4+x^4}$$

$$= \frac{2x}{4+x^4}$$

Hence A is true and R is the correct explanation for A.

Assertion (A): $\int_{-1}^{1} (x^3+\sin x+2)dx = 0$

Reason (R):

$$\int_{-a}^{a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx, & \text{if } f(x) \text{ is an even function} \\ & i.e., (-x) = f(x) \\ 0, & \text{if } f(x) \text{ is an odd function} \\ & i.e., f(-x) = -f(x) \end{cases}$$

Ans. Option (D) is correct.

Explanation:

$$\int_{-a}^{a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx, & \text{if } f(x) \text{ is an even function} \\ & i.e., (-x) = f(x) \\ 0, & \text{if } f(x) \text{ is an odd function} \\ & i.e., f(-x) = -f(x) \end{cases}$$

This is a property of the definite integrals and hence R is true.

$$\int_{-1}^{1} (x^3+\sin x+2)dx$$

$$= \underbrace{\int_{-1}^{1} (x^3+\sin x)dx}_{\text{Odd function}} + \underbrace{\int_{-1}^{1} 2dx}_{\text{Even function}}$$

$$= 0+2[x]_{-1}^{1}$$
$$= 2\times 2$$
$$= 4$$

Hence A is false.